

A Morphing Concept Based on Unsymmetric Composite Laminates and Piezoceramic MFC Actuators

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Bi-stable unsymmetric composite laminates controlled with the piezoelectric MFC actuators are considered as a possible morphing structure. The use of piezoelectric actuators to effect snap-through behavior in bi-stable unsymmetric laminates is studied two ways. First, an experimental actuator-laminate structure is discussed along with efforts to model the shapes and behavior of the structure using the Rayleigh-Ritz technique. The modeling shows promise: the shapes were closely predicted, but the snap-through voltage was under predicted. In the second part of the paper, the capability to model actuator-laminate structures was expanded to include finite-element modeling using the finite-element package ABAQUS. Two finite-element models of simple problems were developed and compared with Rayleigh-Ritz solutions. Agreement between the finite-element and the Rayleigh-Ritz solutions was generally good: the predicted shapes were very similar, but the finite-element solutions were able to capture behavior not seen in the Rayleigh-Ritz solutions.

I. Introduction

There has been considerable attention given to the concept of *morphing* structures, that is, structures that change shape, or state, in response to the environment or in order to change operating characteristics¹⁻⁴. Many morphing concepts rely on the use of hinges, screws, hydraulic actuators, and other such devices to produce changes in the geometry of a structure. Other morphing concepts depend on a continuous supply of power to piezoceramic or shape memory alloy actuators to deform the structure elastically from its natural equilibrium configuration to a 'near-by' configuration. As is well known, a thin, unsymmetrically laminated, elevated-temperature-cure, fiber-reinforced composite laminate, such as a two-layer graphite-epoxy crossply $[0/90]_T$ laminate, can have multiple cylindrical shapes when cooled to room temperature. The laminate can be changed from one cylindrical shape to another by a simple snap-through action by applying moments along opposite edges of the laminate. This multiple-shape phenomenon has been studied by a number of investigators,⁵⁻¹⁰ and is due to the coupling of the residual stresses due to cooling with the geometric nonlinearities due to the large out-of-plane deflections involved. Although this study considers panel-type structures, work has also been done considering unsymmetric laminates as candidates for deployable structures.¹¹ A continuous application of the moments is not required to hold the laminate in either shape; the moments are only required to *change* the configuration of the laminate. Though the crossply laminate is a special case, other laminates have multiple shapes that involve twisting as well as bending curvatures. It is believed that this multiple-shape characteristic could find application in the morphing of structures. The advantage would be that a source of power would only be necessary when changing the shape of the laminate, not to supply the power necessary to hold the laminate in a particular shape. The question is how to effect the shape change, or snap-through. Several researchers^{12,13} have considered the use of shape memory alloy actuators to change the shape. This study considers the use of the NASA Langley Research Center Macro-Fiber Composite Actuator (MFC),^{14,15} an orthotropic piezoceramic actuator, to effect the snap through. The actuator consists of unidirectional piezoceramic fibers embedded in an epoxy matrix and sandwiched between two polyimide films with an interdigitated electrode

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pattern that is used for both poling and actuation; the major direction of actuation is in the direction of the fibers. These actuators are prepackaged and the actuation is accomplished by applying a voltage to the actuator.

This paper is divided into two parts. Considered first is the behavior of an unsymmetric graphite-epoxy crossply $[0/90]_T$ composite laminate with an actuator bonded to one side. Experiments with such an *actuator-laminate structure* are discussed, as are efforts to model such a structure using energy and variational methods and the Rayleigh-Ritz technique. Despite encouraging results using the Rayleigh-Ritz technique, it is felt that a more general and more flexible solution technique must be developed. In the second part of the paper, an attempt is made to improve modeling capabilities by modeling two simple laminate and actuator combinations using the finite-element package ABAQUS;¹⁶ there are no experimental results for these simple combinations.

II. Experimental Work, and Modeling Based on the Rayleigh-Ritz Technique

Considered in this section are an experimental morphing structure and attempts to model the behavior of that structure. In an effort to present this material clearly, the experimental work will be discussed first even though the modeling was used to design the experiment.

A. Experimental Work

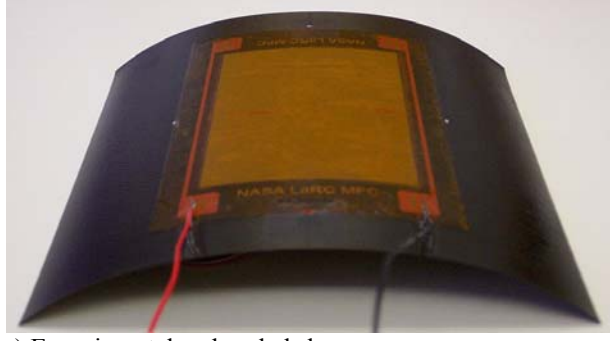
An unsymmetric crossply $[0/90]_T$ composite laminate made from AS4/3502 graphite-epoxy prepreg was cured flat in a pressclave. The laminate was cooled to room temperature, cut to 150 by 150 mm, and the curvatures of the two cylindrical shapes were measured; a slight mismatch in the magnitude of these curvatures was noticed. An MFC actuator with an active portion 86-mm long by 57-mm wide was centrally bonded to one side of the laminate to form what is termed a $[0/90/0^{MFC}]_T$ actuator-laminate structure. To assemble this structure, the laminate was placed on a table so it was concave down and the MFC actuator was positioned on the laminate with the piezoceramic fibers in the direction of the axis of curvature. Epotek 301 low-viscosity two-part epoxy adhesive was applied to the bonding surfaces of the actuator and the laminate, and the assembly was placed inside a vacuum bag. When a vacuum was drawn, the laminate and actuator were brought into contact, deforming both the laminate and the actuator. After the adhesive had cured, the vacuum was removed, the new actuator-laminate structure was removed from the vacuum bag, and the curvatures of the actuator-laminate structure were measured. A strain gage was centrally bonded to the side of the actuator-laminate structure opposite the actuator. This strain gage was positioned to measure strain in the direction transverse to the piezoceramic fibers of the actuator and would be used to determine precisely when the snap-through occurred as voltage to the actuator was increased. The assembly was kept in a desiccator until experiments were conducted. The actuator-laminate structure, in both stable equilibrium shapes is seen in Figs. 1a and b. The shape of the actuator-laminate structure as it was in the vacuum bag was termed the *as-bonded shape* and is seen in Fig. 1a, whereas the other shape, seen in Fig 1b was termed the *second stable shape* of the actuator-laminate structure. The as-bonded shape is qualitatively similar to the unbonded shape of the laminate, albeit with slightly less curvature. However, the second stable shape of the actuator-laminate structure is considerably different from the unbonded second shape of the laminate; the shape is much shallower and, in the center of the structure, the direction of curvature is opposite the global direction of curvature of the structure. That is, there is a bulge under the actuator in the second stable shape.

The experimental setup consisted of a voltage supply, voltage amplifier, strain gage amplifier/conditioner, voltmeter, a LabVIEW data acquisition system on a personal computer, and the actuator-laminate structure with the strain gage bonded to the side opposite the actuator. Since, as will be discussed, the analyses of the model assumed the edges of the actuator-laminate structure were free of any specified force resultants or displacements, the actuator-laminate structure in the shape of Fig. 1b was suspended, like a pendulum, by the strain gage wire when conducting the snap-through experiments. The voltage was increased from zero until the actuator-laminate structure snapped to a shape similar to the shape of Fig. 1a; the voltage was then decreased to zero, and the actuator-laminate structure returned to the as-bonded shape (Fig. 1a). This procedure was repeated six times with good repeatability, and an average snap-through voltage of 1695 V, and with a standard deviation of 47 V.

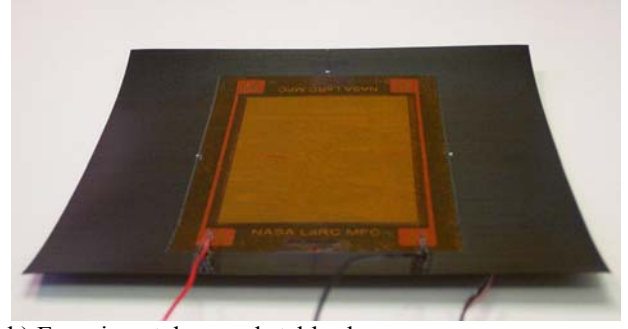
B. Modeling Based on the Rayleigh-Ritz Technique

1. Overview

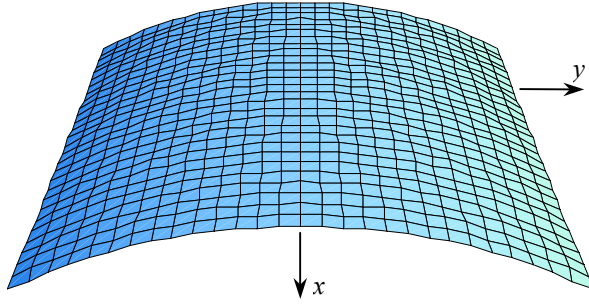
Three models of the above-described experiment that are based on the Rayleigh-Ritz technique are considered in this section. With the Rayleigh-Ritz technique, displacement fields are assumed and stationary values of the total potential energy are sought; these stationary values represent equilibrium shapes. It will be seen that the equations for the total potential energy have only internal energy terms because it is assumed that there are no external loads on applied to the actuator-laminate structure. The three models considered in this section are very similar, but each



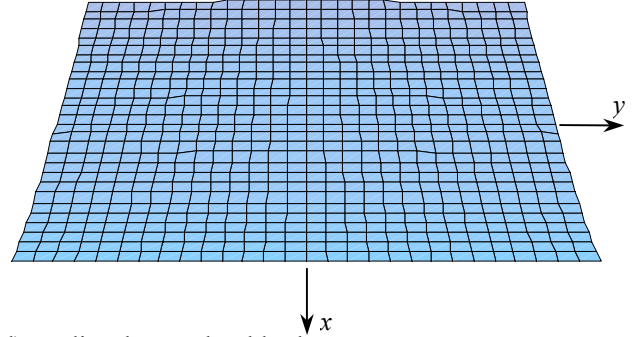
a) Experimental as-bonded shape



b) Experimental second stable shape



c) Predicted as-bonded shape



d) Predicted second stable shape

Figure 1 Experimental and predicted stable shapes of actuator-laminate structure with zero applied voltage

considers a different set of displacement fields. The displacement fields are formulated so the structure is fixed at the origin, the geometric center of the laminate. The coordinate system is as shown in Figs. 1c and d. The three models were programmed using the technical computing software Mathematica.¹⁷

As with the application of energy principles to any problem, stable equilibrium shapes are found by minimizing the total potential energy of the laminate, or actuator-laminate structure, depending on which is being considered. Though the shapes of the cooled laminate without an actuator and the change in shapes of the cooled laminate due to bonding on the actuator are of interest, the shapes of the actuator-laminate structure when voltage is applied to the actuator are of primary interest, so the question reduces to determining what shape or shapes renders the total potential energy of the actuator-laminate structure a minimum at specified levels of the applied voltage. It is assumed that the laminate obeys the Kirchhoff hypothesis, and that hypothesis is used in conjunction with the von Kármán approximation to the geometrically nonlinear strain-displacement relations:

$$\begin{aligned}\varepsilon_x^0 &= \frac{\partial u^0}{\partial x} + \frac{1}{2} \left(\frac{\partial w^0}{\partial x} \right)^2 \\ \varepsilon_y^0 &= \frac{\partial v^0}{\partial y} + \frac{1}{2} \left(\frac{\partial w^0}{\partial y} \right)^2 \\ \gamma_{xy}^0 &= \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w^0}{\partial x} \frac{\partial w^0}{\partial y}\end{aligned}\tag{1}$$

where u^0 and v^0 are the x - and y -reference-surface displacements, w^0 is the z -displacement, and the ε^0 s and γ_{xy}^0 are the strains at the reference surface. Throughout the modeling, the reference surface is taken to be the midplane of the laminate. The model is developed in three steps: cooling the laminate, bonding the actuator to the laminate, and activating the actuator as part of the actuator-laminate structure. In each step, the first variation of the total potential energy is used to determine conditions that render the energy stationary; the second variation is used to determine the stability of the stationary conditions. The result of applying the Rayleigh-Ritz and variational approaches to each

step is a set of nonlinear algebraic equations that predict the shape of the laminate. The shapes can be solved for as a function of increasing voltage applied to the actuator. The relationship between the shapes and the voltage is multivalued in the area of interest and exhibits limit-point behavior. The existence of the limit-point behavior is important because it is interpreted to mean that when the voltage reaches a certain value on a certain branch, the voltage-laminate-shape relationship cannot continue on that same solution branch, rather it must jump, i.e., ‘snap through,’ to another solution branch. This is an indication that the laminate will change shape at that voltage level.

In the first step, cooling the laminate from the cure temperature, the actuator is not considered and the total potential energy, Π , is given by

$$\Pi = \Pi_1 = \frac{1}{2} \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \int_{z_0}^{z_2} [(\sigma_x - \sigma_x^T) \varepsilon_x + (\sigma_y - \sigma_y^T) \varepsilon_y + (\sigma_{xy} - \sigma_{xy}^T) \gamma_{xy}] dx dy dz \quad (2)$$

where L_x and L_y are the laminate sidelengths in the x - and y -directions, respectively, and z_2 and z_0 are the z -positions of the top and bottom surfaces of the laminate, respectively. The σ s are the stresses and are given by

$$\begin{aligned} \sigma_x &= \bar{Q}_{11} \varepsilon_x + \bar{Q}_{12} \varepsilon_y + \bar{Q}_{16} \gamma_{xy} - \sigma_x^T \\ \sigma_y &= \bar{Q}_{12} \varepsilon_x + \bar{Q}_{22} \varepsilon_y + \bar{Q}_{26} \gamma_{xy} - \sigma_y^T \\ \sigma_{xy} &= \bar{Q}_{16} \varepsilon_x + \bar{Q}_{26} \varepsilon_y + \bar{Q}_{66} \gamma_{xy} - \sigma_{xy}^T \end{aligned} \quad (3)$$

and the σ^T s are the thermally-induced stresses:

$$\begin{aligned} \sigma_x^T &= \bar{Q}_{11} \varepsilon_x^T + \bar{Q}_{12} \varepsilon_y^T + \bar{Q}_{16} \gamma_{xy}^T \\ \sigma_y^T &= \bar{Q}_{12} \varepsilon_x^T + \bar{Q}_{22} \varepsilon_y^T + \bar{Q}_{26} \gamma_{xy}^T \\ \sigma_{xy}^T &= \bar{Q}_{16} \varepsilon_x^T + \bar{Q}_{26} \varepsilon_y^T + \bar{Q}_{66} \gamma_{xy}^T \end{aligned} \quad (4)$$

The $\bar{Q}$ s are the transformed reduced stiffness and are defined as usual, and the ε^T s and γ_{xy}^T are the free-thermal strains and are given by

$$\begin{aligned} \varepsilon_x^T &= \alpha_x \Delta T \\ \varepsilon_y^T &= \alpha_y \Delta T \\ \gamma_{xy}^T &= \alpha_{xy} \Delta T = 0 \end{aligned} \quad (5)$$

where the α s are the coefficients of thermal deformation and ΔT is the change in temperature from the cure temperature.

The second step models the bonding of the actuator to the laminate and is formulated to allow slipping between the actuator and the laminate, as could occur experimentally before the bonding adhesive had cured. There will be a through-thickness strain discontinuity between the actuator and laminate due to the processing of the laminate and the subsequent bonding process. In this step, both the laminate and the actuator are considered, so the total potential energy is the sum of the total potential energy of the laminate, Π_1 , and the total potential energy of the actuator, Π_2 :

$$\Pi_2 = \frac{1}{2} \int_{-\frac{L_x^{MFC}}{2}}^{\frac{L_x^{MFC}}{2}} \int_{-\frac{L_y^{MFC}}{2}}^{\frac{L_y^{MFC}}{2}} \int_{z_2}^{z_3} [\sigma_x^{a2} \varepsilon_x^{a2} + \sigma_y^{a2} \varepsilon_y^{a2} + \sigma_{xy}^{a2} \gamma_{xy}^{a2}] dx dy dz \quad (6)$$

where L_x^{MFC} and L_y^{MFC} are the actuator sidelengths in the x - and y -directions, respectively, z_3 is the z -position of the top surface of the actuator, z_2 is the z -position of the interface between the actuator and the laminate, and the σ^{a2} s are stresses in the actuator and are given by

$$\begin{aligned}\sigma_x^{a2} &= \bar{Q}_{11}\varepsilon_x^{a2} + \bar{Q}_{12}\varepsilon_y^{a2} + \bar{Q}_{16}\gamma_{xy}^{a2} \\ \sigma_y^{a2} &= \bar{Q}_{12}\varepsilon_x^{a2} + \bar{Q}_{22}\varepsilon_y^{a2} + \bar{Q}_{26}\gamma_{xy}^{a2} \\ \sigma_{xy}^{a2} &= \bar{Q}_{16}\varepsilon_x^{a2} + \bar{Q}_{26}\varepsilon_y^{a2} + \bar{Q}_{66}\gamma_{xy}^{a2}\end{aligned}\quad (7)$$

and the ε^{a2} s and γ_{xy}^{a2} are strains in the actuator, with the simplification that the curvatures at the midsurface of the actuator are the same as at the reference surface, that are thus given by

$$\begin{aligned}\varepsilon_x^{a2} &= \left(z - \left(\frac{z_3 - z_0}{2} \right) \right) \kappa_x^\theta \\ \varepsilon_y^{a2} &= \left(z - \left(\frac{z_3 - z_0}{2} \right) \right) \kappa_y^\theta \\ \gamma_{xy}^{a2} &= \left(z - \left(\frac{z_3 - z_0}{2} \right) \right) \kappa_{xy}^\theta = 0\end{aligned}\quad (8)$$

The κ^θ s are the *reference-surface* curvatures, i.e., the curvatures at the midplane of the laminate.

In the third step of the modeling, the actuation of the actuator-laminate structure is considered. In this step, the bond between the actuator and the laminate has cured so there is no longer slipping between the actuator and the laminate; the through-thickness strain discontinuity is now fixed in value. Once again, the total potential energy is the sum of Π_1 and the total potential energy of the actuator, now Π_3 :

$$\Pi_3 = \frac{1}{2} \int_{-\frac{L_x^{MFC}}{2}}^{\frac{L_x^{MFC}}{2}} \int_{-\frac{L_y^{MFC}}{2}}^{\frac{L_y^{MFC}}{2}} \int_{z_2}^{z_3} [(\sigma_x^{a3} - \sigma_x^E) \varepsilon_x^{a3} + (\sigma_y^{a3} - \sigma_y^E) \varepsilon_y^{a3} + (\sigma_{xy}^{a3} - \sigma_{xy}^E) \gamma_{xy}^{a3}] dx dy dz \quad (9)$$

where the ε^{a3} s and γ_{xy}^{a3} are strains in the actuator given and will be discussed below. The σ^{a3} s are the stresses in the actuator and are given by

$$\begin{aligned}\sigma_x^{a3} &= \bar{Q}_{11}\varepsilon_x^{a3} + \bar{Q}_{12}\varepsilon_y^{a3} + \bar{Q}_{16}\gamma_{xy}^{a3} - \sigma_x^E \\ \sigma_y^{a3} &= \bar{Q}_{12}\varepsilon_x^{a3} + \bar{Q}_{22}\varepsilon_y^{a3} + \bar{Q}_{26}\gamma_{xy}^{a3} - \sigma_y^E \\ \sigma_{xy}^{a3} &= \bar{Q}_{16}\varepsilon_x^{a3} + \bar{Q}_{26}\varepsilon_y^{a3} + \bar{Q}_{66}\gamma_{xy}^{a3} - \sigma_{xy}^E\end{aligned}\quad (10)$$

The σ^E s are the piezoelectrically-induced stresses and are given by

$$\begin{aligned}\sigma_x^E &= \bar{Q}_{11}\varepsilon_x^E + \bar{Q}_{12}\varepsilon_y^E + \bar{Q}_{16}\gamma_{xy}^E \\ \sigma_y^E &= \bar{Q}_{12}\varepsilon_x^E + \bar{Q}_{22}\varepsilon_y^E + \bar{Q}_{26}\gamma_{xy}^E \\ \sigma_{xy}^E &= \bar{Q}_{16}\varepsilon_x^E + \bar{Q}_{26}\varepsilon_y^E + \bar{Q}_{66}\gamma_{xy}^E\end{aligned}\quad (11)$$

where the ε^E s and γ_{xy}^E are piezoelectrically-induced strains. Considering the actuator as an orthotropic linear piezoelectric plate, the piezoelectrically-induced strains are given by

$$\begin{aligned}
\varepsilon_x^E &= d_{11} \frac{\Delta V}{\Delta x_1} \\
\varepsilon_y^E &= d_{12} \frac{\Delta V}{\Delta x_1} \\
\gamma_{xy}^E &= 0
\end{aligned} \tag{12}$$

where the ds are effective piezoelectric coefficients, ΔV is the voltage applied to the MFC actuator and Δx_1 is the spacing of the interdigitated electrodes on the actuator. The strains in the actuator, ε^{a3} s and γ_{xy}^{a3} , are given by

$$\begin{aligned}
\varepsilon_x^{a3} &= \varepsilon_x - \varepsilon_x^s \\
\varepsilon_y^{a3} &= \varepsilon_y - \varepsilon_y^s \\
\gamma_{xy}^{a3} &= \gamma_{xy} - \gamma_{xy}^s
\end{aligned} \tag{13}$$

where the ε s and γ_{xy} are Kirchhoff-like strains that are zero at the reference surface (i.e., at the midplane of the laminate) and vary linearly with thickness. The ε^s s and γ_{xy}^s are considered *shift strains*, account for the discontinuous through-thickness strains due to the room-temperature bonding of the actuator to the laminate, and allow the strain in the actuator to be defined by reference-surface strains and curvatures. The shift strains are given by

$$\begin{aligned}
\varepsilon_x^s &= \varepsilon_x^{i0} + \left(\frac{z_3 - z_0}{2} \right) \kappa_x^{i0} \\
\varepsilon_y^s &= \varepsilon_y^{i0} + \left(\frac{z_3 - z_0}{2} \right) \kappa_y^{i0} \\
\gamma_{xy}^s &= \gamma_{xy}^{i0} + \left(\frac{z_3 - z_0}{2} \right) \kappa_{xy}^{i0}
\end{aligned} \tag{14}$$

where the ε^{i0} s, γ_{xy}^{i0} , and κ^{i0} s are the midplane strains and curvatures in the as-bonded shape. It should be noted that the formulation of step three of the model (Eqs. (9)-(14)) is a correction of the formulation from the authors' previous publications. However, using this corrected version, identical shapes and snap-through voltages are obtained because the difference between this and the earlier version is a constant that is eliminated when finding stationary values of the total potential energy.

2. Displacement Fields

Three different displacement fields were used in the formulation represented by Eqs. (1)-(14) to create three different models. These displacement fields were chosen with the hope that they would closely resemble the actual shapes of the actuator-laminate structure, a requirement for good results from the Rayleigh-Ritz technique. The three sets of displacement fields increase in complexity and have four, ten, or sixteen unknown terms. These unknown terms are what are varied in order to find stationary values of the total potential energy. The to-be-determined terms in each model are c_i , where i varies from 1 to the number of terms in the model (i.e., 4, 10, or 16).

The four-term displacement fields,

$$\begin{aligned}
u^0 &= c_3 x - \frac{1}{6} c_1^2 x - \frac{1}{4} c_1 c_2 x y^2 \\
v^0 &= c_4 x - \frac{1}{6} c_2^2 x - \frac{1}{4} c_1 c_2 x^2 y \\
w^0 &= \frac{1}{2} (c_1 x^2 + c_2 y^2)
\end{aligned} \tag{15}$$

have been successfully used in the past to model the room-temperature shapes of elevated cure crossply laminates.⁹ However, the w^0 -displacement field in Eq. (15) could not capture the reverse curvature in center of the laminate in the second stable shape (Fig. 1b). In response, the ten-term model was created to account for this area of reverse curvature by using both second- and third-order terms in the assumed w^0 -displacement field:

$$\begin{aligned} u^0 &= c_5x + \frac{1}{3}c_6x^3 + c_7xy^2 - \frac{1}{6}c_1^2x^3 - \frac{1}{8}c_1c_3x^4 - \frac{1}{40}c_3^2x^5 \\ v^0 &= c_8y + c_9x^2y + \frac{1}{3}c_{10}y^3 - \frac{1}{6}c_2^2y^3 - \frac{1}{8}c_2c_4y^4 - \frac{1}{40}c_4^2y^5 \\ w^0 &= \frac{1}{2}(c_1x^2 + c_2y^2) + \frac{1}{6}(c_3x^3 + c_3y^3) \end{aligned} \quad (16)$$

It should be noted that, using the w^0 -displacement field of Eq. (16), the displacements over the entire actuator-laminate structure cannot be modeled. Instead, a quarter symmetry argument is invoked (only the first quadrant of the actuator-laminate structure is considered) and Eqs. (2), (6), and (9) are adjusted accordingly. Once the assumed w^0 displacement was chosen, complete polynomials of order three were chosen for the strain in the x and y directions. Next, terms were removed in order to match the boundary and matching conditions. However, in order to easily solve the nonlinear equations that result in stationary values of the total potential energy, some additional simplifications were made that reduced the number of terms from sixteen to ten. As might be expected, the w^0 -displacement field of the sixteen-term model is the same as that of the ten-term model, and the u^0 - and v^0 -displacement fields are without the simplifications that were made for the ten-term model:

$$\begin{aligned} u^0 &= c_5x + \frac{1}{2}c_6x^2 + c_7xy + \frac{1}{3}c_8x^3 + \frac{1}{2}c_9x^2y + c_{10}xy^2 - \frac{1}{6}c_1^2x^3 - \frac{1}{8}c_1c_3x^4 - \frac{1}{40}c_3^2x^5 \\ v^0 &= c_{11}y + c_{12}xy + \frac{1}{2}c_{13}y^2 + c_{14}x^2y + \frac{1}{2}c_{15}xy^2 + \frac{1}{3}c_{16}y^3 - \frac{1}{6}c_2^2y^3 - \frac{1}{8}c_2c_4y^4 - \frac{1}{40}c_4^2y^5 \\ w^0 &= \frac{1}{2}(c_1x^2 + c_2y^2) + \frac{1}{6}(c_3x^3 + c_3y^3) \end{aligned} \quad (17)$$

With these displacement fields, the total potential energy, Π , for the laminate or actuator-laminate structure, depending on which step of the modeling is being considered, can be written as a function of the to-be-determined terms, and finding the stationary values of Π reduces to solving the following system of nonlinear algebraic equations:

$$\frac{\partial \Pi}{\partial c_i} = 0 \quad i = 1, n \quad (18)$$

where $n = 4, 10$, or 16 depending on whether the four-, ten-, or sixteen-term displacement fields are being used, respectively.

3. Comparison between experiment and models

As mentioned earlier, the average snap-through voltage from the experiments was 1695 V. Discussed next will be the predictions of the models, in particular, the shapes of the actuator-laminate structure and how they change with the application of a voltage to the MFC actuator. The first step was to compare the actual and predicted shapes of the laminate without the actuator with the measurements that were made on the actual laminate. Measurements of the actual laminate showed that the laminate relaxed over time (i.e., the curvatures of the two shapes decreased over time) and that the magnitude of the curvatures of the two stable shapes was slightly different. In order to match the predictions of the average curvature of the actual laminate the ΔT in the models was adjusted. It was assumed that the slight mismatch in the magnitude of the curvatures was due to uneven resin bleed during manufacture, so the thicknesses and material properties were changed to capture this effect. The MFC actuator used in this study was of an older design with somewhat unknown characteristics, so the effective piezoelectric coefficients were determined by bonding another actuator (of the same design) to a cantilevered aluminum plate and measuring the tip displacement of plate as voltage was applied to the actuator. The material properties and other data that were

required in the models are given in Table 1.

Figure 2 represents the shapes of the actuator-laminate structure and their variation with the application of voltage; specifically, it shows the variation in the w^0 displacement of the corners of the laminate (as part of the actuator-laminate structure) with applied voltage. For comparison, the predictions of the four-, ten-, and sixteen-term models are all shown. It is seen that at 0 V, the models predict three possible w^0 displacements, which correspond to three equilibrium shapes of the actuator-laminate structure. For example, following the prediction of the four-term model, point 1 represents the as-bonded shape (Fig. 1a) and point 3 represents the second stable shape (Fig. 1b) of the actuator-laminate structure. Point 2 represents a third equilibrium shape, one that is not physically realizable because it is an unstable shape. To effect snap through (again following the four-term model), starting from point 3, the applied voltage is increased. The shape of the structure changes little until point 5 is reached, whereupon further increase in voltage will cause the structure to suddenly snap through to point 4. From point 4, a reduction in the applied voltage will bring the structure back to the as-bonded shape represented by point 1. Because no limit point is seen for negative applied voltage, the structure cannot be transformed from point 1 back to point 3 through the application of voltage to the MFC actuator. The four-term model predicts a snap-through voltage of 771 V, the ten-term model predicts 1262 V, and the sixteen-term model predicts 822 V. There are significant differences in these predicted snap-through voltages, and all are less than the average experimental snap-through voltage of 1695 V. When compared, both qualitatively and quantitatively, with the experimental actuator-laminate structure, all

Table 1 Layer Properties

Property	AS4/3502 (Layer 1)	AS4/3502 (Layer 2)	MFC Actuator (active portion)
E_1 (GPa)	132.0	128.0	29
E_2 (GPa)	9.798	9.608	18
G_{12} (GPa)	5.112	4.895	5.3
ν_{12}	0.2990	0.3009	0.28
α_1 (1/°C)	-0.04156×10^{-6}	5.538×10^{-9}	
α_2 (1/°C)	23.77×10^{-6}	24.78×10^{-6}	
d_{11} ($\mu\text{C}/(\text{kV}/\text{mm})$)*			281
d_{12} ($\mu\text{C}/(\text{kV}/\text{mm})$)*			-111
thickness (m)	133.8×10^{-6}	138.2×10^{-6}	$290. \times 10^{-6}$
L_x (m)	0.1518		0.08573
L_y (m)	0.1518		0.05715
Δx_1 (m)			1.0668×10^{-3}
ΔT (°C)	-117		

*determined by calibration with cantilevered aluminum plate

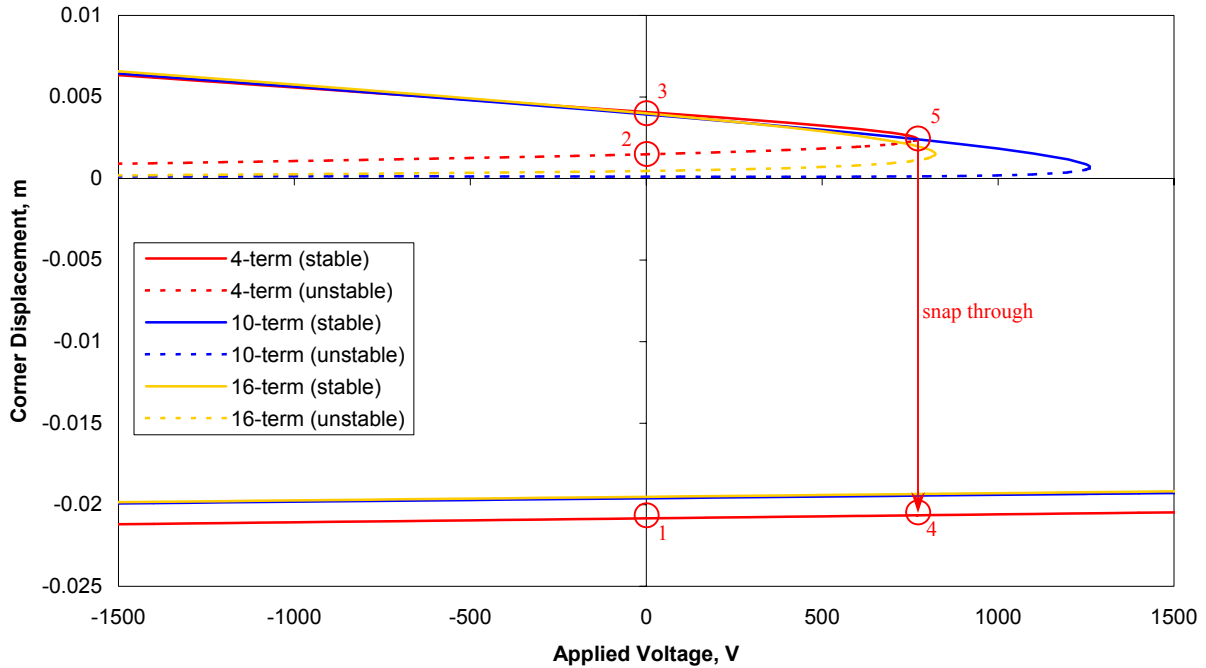


Figure 2 Corner displacement, w^0 , of the $[0/90/0]_{\text{T}}^{\text{MFC}}$ actuator-laminate structure as a function of voltage applied to the MFC actuator

three models do a good job of predicting the shapes of the structure. In particular, the ten- and sixteen-term models (which predict almost identical w^0 displacements) capture the central region of reverse curvature seen in Fig 1b. Figures 1c and d show the stable zero-voltage shapes predicted using the ten-term model.

Figure 3 allows the examination of the snap-through event in a different way. Figure 3 shows the y -direction strain, ϵ_y , of the bottom of the actuator-laminate structure (where the experimental strain gage was) as the voltage is varied for a representative experimental case and for the model predictions. The structure begins in the second stable configuration (point 3) where the strain is set to zero. Again following the four-term model, as the voltage is increased, the strain changes little until point 5 is reached where the strain suddenly drops to point 4; the snap through has occurred. Reduction in applied voltage brings the structure back to the as-bonded shape of Fig 1a. It is interesting to note that the snap-through voltage appears to be directly proportional to the magnitude of the strain in the as-bonded configuration (point 1); the implication may be that capturing the in-plane strains with the assumed displacement fields may be as important as capturing the out-of-plane displacement. Although not shown here, the experimental cases, which did not end at exactly the same strain each time, also showed this trend.

III. Finite-Element Modeling

Although interesting and encouraging results were found using the Rayleigh-Ritz technique, it is felt that a more general solution technique must be developed to analyze laminates with other than crossply stacking sequences, and structures with more interesting geometries. In addition, it was felt that the displacement fields that were used for the Rayleigh-Ritz modeling may not have been sufficient to capture all of the local deformations, and increasing the complexity of the displacement fields quickly increases the difficulty in solving the set of nonlinear equations for the equilibrium shapes. Therefore, in this section, two relatively simple problems are discussed and models of these problems are developed using finite-element methods. The main purpose of these problems is to begin working toward improving modeling capabilities for problems like the actuator-laminate structure discussed in section II, and then actuator-laminate structures with non-crossply laminates and more complicated geometries. The problems considered are like simplified versions of the problem that was discussed in section II: they have no external loads, the same central displacement constraint, and square planform geometry. Although the planform geometry of the actuator-laminate structure that was discussed in section II is very simple, there are some elements that increase the difficulty of modeling such a structure. Among these elements are the orthotropy of the MFC actuator, the partial coverage of the laminate by the actuator, the assumed difference in the layer thickness and material properties of the graphite-epoxy layers, and bonding the actuator to the laminate at room temperature. The first problem considered in

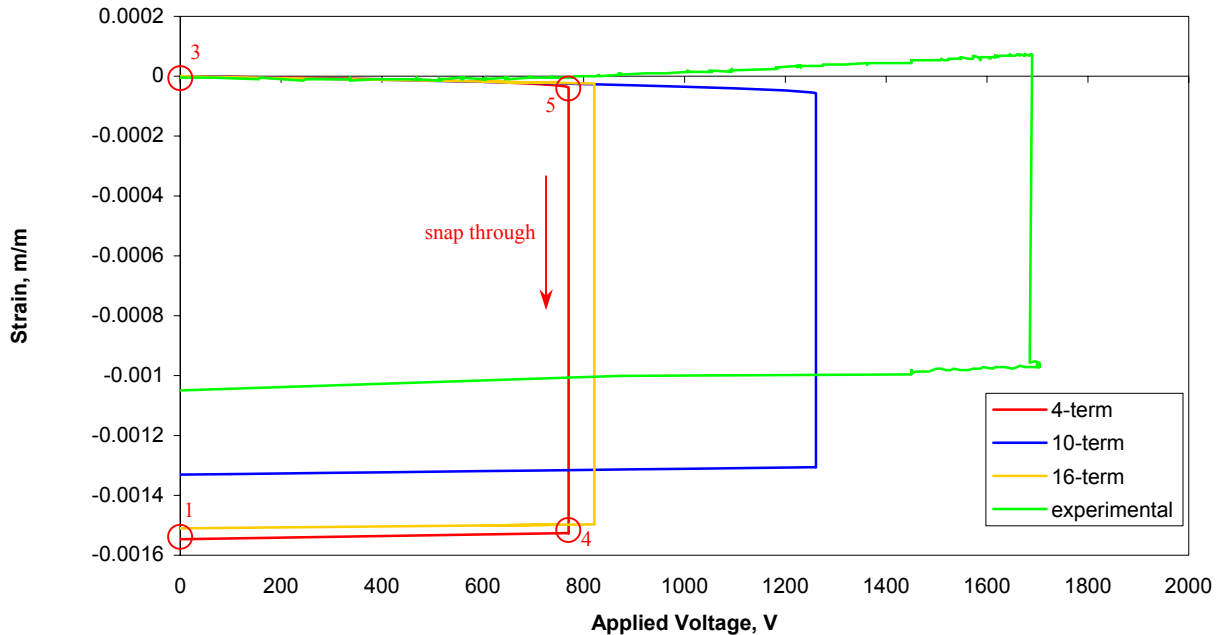


Figure 3 Strain, ϵ_y , of the bottom center of the actuator-laminate structure as a function of voltage applied to the MFC actuator

this section has none of these complicating elements, and the second problem has only one. For both of the problems, a model created using the finite-element package ABAQUS is used and compared with Rayleigh-Ritz models similar to those used in the previous section. Because they are less costly than solid elements, shell elements were used in the ABAQUS models. However, there are no piezoelectric shell elements in ABAQUS so the piezoelectric effect was modeled using the thermal deformation capabilities of ABAQUS; this decouples the electrical and mechanical piezoelectric effects. In order to capture the entire actuation curves in the ABAQUS models, the modified Riks solution capability of ABAQUS is used. In a Riks solution, the load is a variable and is solved for along with the displacements. In both problems, somewhat unrealistic dimensions were chosen so that desired behavior could be captured. Rayleigh-Ritz models, very much like those discussed in section II, using the four-term and the sixteen-term displacement fields, were used to compare results with the finite-element models.

Table 2 Layer Properties

Property	Graphite-Epoxy	PZT
E_1 (GPa)	130.0	69.0
E_2 (GPa)	9.701	69.0
G_{12} (GPa)	5.0	
G_{13} (GPa)	5.0	
G_{23} (GPa)	5.0	
ν_{12}	0.3	0.3
α_1 (1/°C)	-0.018×10^{-6}	2.0×10^{-6}
α_2 (1/°C)	24.282×10^{-6}	2.0×10^{-6}
d_{31} (m/V)		-122×10^{-12}
thickness (m)	125×10^{-6}	125×10^{-6}

A. [PZT/90₂/0₂/PZT]_T Actuator-Laminate Structure

The first problem considered consists of a square, 0.25 m by 0.25 m, [90₂/0₂]_T graphite-epoxy laminate with a layer of the piezoceramic PZT on both sides of the laminate. It is assumed that the PZT layers are sized to cover the entire laminate and are bonded to the laminate at the cure temperature while the laminate is flat. The assumed material properties of the graphite-epoxy and PZT layers are given in Table 2. Because the PZT layers are considered isotropic, both the thermal and piezoelectric effects could be modeled using the thermal deformation capabilities of ABAQUS; the PZT layers are cooled with the laminate, and then the temperature of the PZT layers is varied from the cooled temperature to represent the actuation. (This method for cooling and actuation would not work with an anisotropic layer because the ratio of the deformation in the x - and y -directions would not be the same for the thermal and piezoelectric deformations.) During actuation, in order to have the two PZT layers work together, the applied electric fields in the top and bottom PZT layers were set to be equal and opposite.

Because the PZT layers needed to be cooled, then actuated, and the graphite-epoxy layers were only cooled, the model was split into three layers of elements: the bottom PZT layer, the base [90₂/0₂]_T laminate, and the top PZT layer. In order to form a complete structure, the three layers were tied together using a surface-based tie constraint that, as used here, makes all of the degrees of freedom of the two surfaces equal. The ABAQUS model of this problem was built using linear plane stress shell (S4) elements and a 10-element by 10-element mesh for each layer. Rayleigh-Ritz models, very much like those discussed in section II, using the four-term and the sixteen-term displacement fields, were used to compare results with the finite-element models.

To represent the shapes and actuation of this actuator-laminate structure, Fig. 4 shows the out-of-plane displacement of the corner of the structure, w^0 ($L_x/2$, $L_y/2$), plotted vs. the piezoelectrically-induced strain in the bottom layer of PZT, which would be caused by a through-thickness electric field in that layer. (Recall that the electric field in the top PZT layer is equal and opposite that in the bottom PZT layer.) The results from the ABAQUS model (blue icons) are shown along with results from four-term (red lines) and sixteen-term (gold lines) Rayleigh-Ritz models. As in section II, with no actuation, the actuator-laminate structure has three equilibrium shapes; two of these shapes are stable and the other is unstable. However, unlike section II, the displacement vs. piezoelectrically-induced strain relations are continuous in addition to being multivalued for all values. That is, each stable branch of the displacement vs. induced strain relations is connected to the unstable branch; the structure shows the same type of behavior for positive and negative piezoelectrically-induced strains because the model considered in this section is perfectly antisymmetric. It is seen that at zero induced strain the models have good agreement, though the ABAQUS displacements are slightly smaller. However, although not shown, the ABAQUS results are much closer for the midside w^0 -displacements (at $(L_x/2, 0)$) indicating that overall the shapes are closer than Figure 4 would suggest. On the other hand, the limit points (i.e., the snapping induced strain) for the three analyses are significantly different. It is seen that in the cases of both the w^0 -displacement and the limit points, the sixteen-term Rayleigh-Ritz predictions are closer than the four-term predictions to the ABAQUS results. It should also be noted that the ABAQUS model was also run with a 20-element-by-20-element mesh for each layer, reduced integration (S4R) elements, quadratic (S9R5) elements, and with very stiff through-thickness shear moduli (G_{13} , G_{23}), and all changes had negligible effects on the results.

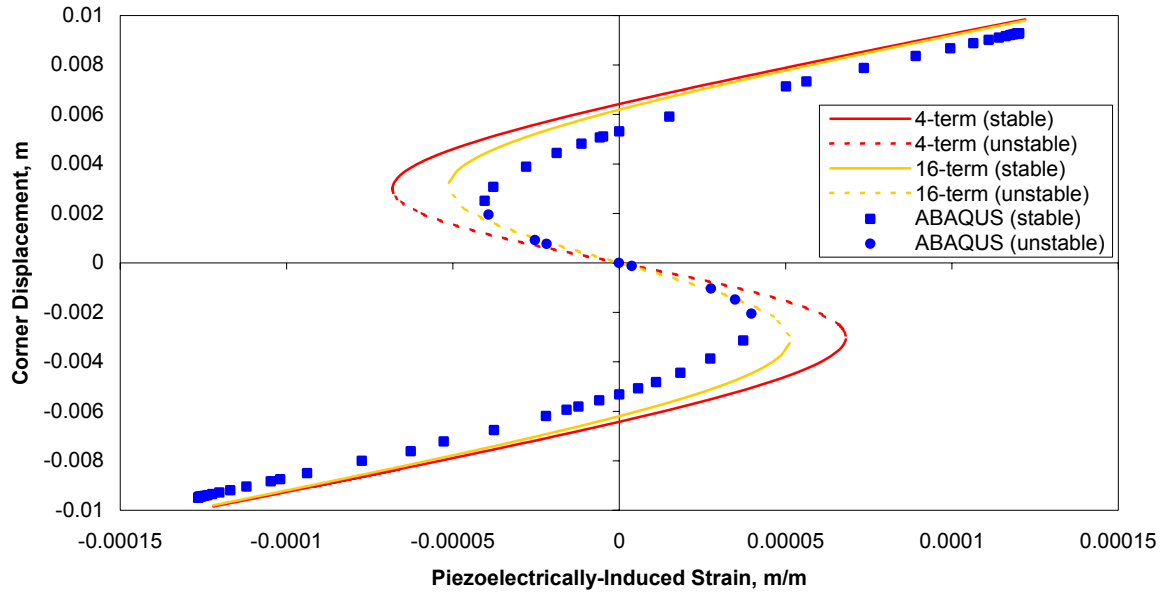


Figure 4 Corner displacement, w^0 , of the $[PZT/90_2/0_2/PZT]_T$ actuator-laminate structure as a function of the piezoelectrically-induced strain, ϵ^E , in the bottom PZT layer

B. $[0/90/0^{MFC}]_T$ Actuator-Laminate Structure

The second actuator-laminate structure considered in this section is very similar to the structure that was considered in section II (i.e., a graphite-epoxy laminate with an MFC actuator bonded to one side) with one major exception—the MFC actuator covers the entire base laminate rather than just a central portion. So the actuator-laminate structure is a $[0/90/0^{MFC}]_T$ laminate consisting of all full layers. As in section II, the laminate is manufactured, and then, at room temperature, the actuator and laminate are bonded together. The material properties for the graphite-epoxy are given in Table 2 and the MFC actuator properties are given in Table 1. (The results shown here used the assumption that $G_{12}^{MFC} = G_{13}^{MFC} = G_{23}^{MFC}$; however, the results have been shown to be insensitive to the through-thickness shear moduli).

Because the laminate and the actuator begin apart and are joined later, the ABAQUS surface-based tie constraint used in the section IIIA cannot be used. Instead, frictional interactions were used to join the actuator to the laminate. The model consisted of two layers of shell elements, one layer for the $[0/90]_T$ laminate and one layer for the MFC actuator, with the reference surfaces offset from the midplanes so that they were in contact with each other. Each layer had a mesh of 10 elements by 10 elements. At the beginning, the actuator is touching the laminate with the surface condition of no separation so that the surfaces are constrained to stay in contact, but with no friction between them. Moments are applied to two edges of the laminate to force the laminate into the desired cylindrical shape, a temperature change is applied to the laminate to simulate the cooling from the cure temperature, and then the moments are removed from the laminate. Because of the no separation condition, the actuator has deformed with the laminate, and because of the no friction condition, slipping could occur between the actuator and the laminate so, at this point, the actuator and laminate are like they are in the vacuum bag before the bonding adhesive has cured, like the end of step II of the Rayleigh-Ritz models. The friction between the actuator and laminate is then changed to ‘rough’ friction, which prevents all slipping between surfaces; the shape does not change and it is like the bonding adhesive has now cured to form the actuator-laminate structure. (One possible concern about this approach is that, although the translational degrees of freedom are constrained between the actuator and laminate layers, the rotational degrees of freedom are not.) With the full-coverage MFC layer, the as-bonded shape is the unactuated shape for the structure. There is only one possible shape at zero applied voltage. However, simulated actuation predicts that other shapes may be possible with this type of structure. Figure 5 shows the corner w^0 displacement as a function of the fiber-direction piezoelectrically-induced strain of the MFC actuator. The as-bonded shape is represented at zero induced strain. If the induced strain is negatively increased (e.g., by negatively increasing the applied voltage), eventually the three equilibrium shapes emerge. In this case, the predictions of displacement seen in Fig. 5 are very similar for all of the models. The major difference is that ABAQUS predicts a limit point, and

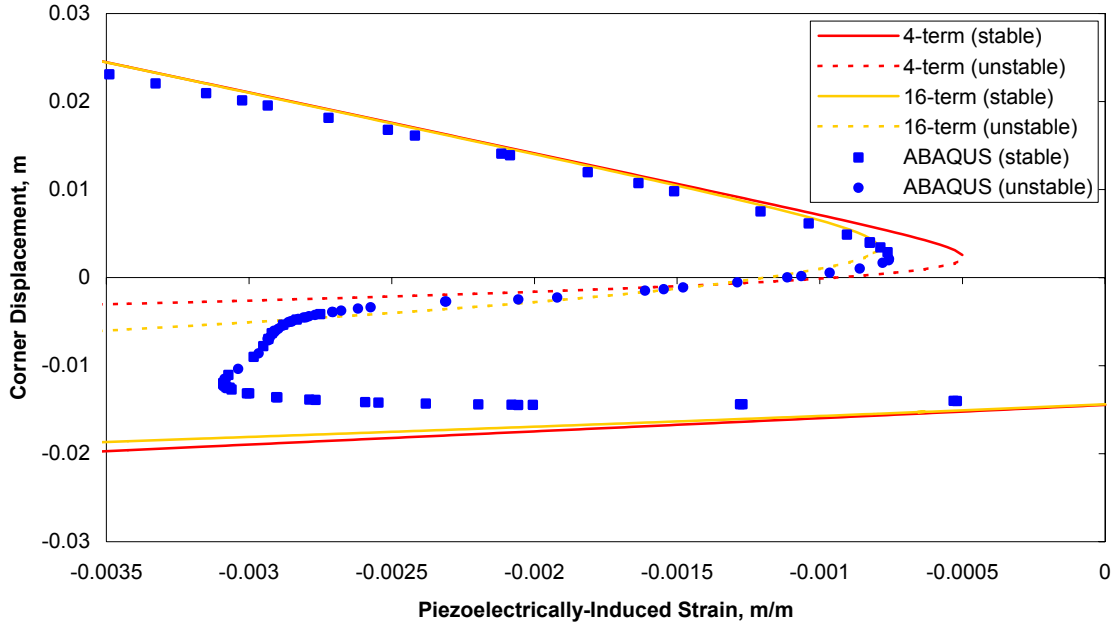


Figure 5 Corner displacement, w^0 , of the $[0/90/0]_T^{\text{MFC}}$ actuator-laminate structure as a function of the fiber-direction piezoelectrically-induced strain, ϵ_x^E , in the MFC actuator

therefore a chance for snap-through behavior to occur, as the induced strain is negatively increased from zero. From reviewing the ABAQUS results, it appears that smaller, more localized deformations appear near where ABAQUS predicts this limit point. In particular, it appears that there is significant shear strain near the corners and, rather than a single bulge in the center of the structure, a more complicated out-of-plane displacement. The ability to capture behavior like this, behavior that is too complicated for the current Rayleigh-Ritz models is one reason for exploring the use of finite-element methods.

IV. Conclusions

Two aspects of the use of piezoelectric actuators to effect snap-through behavior in bi-stable unsymmetric composite laminates were considered. First, an unsymmetric laminate was manufactured, an MFC actuator was bonded to it, and this actuator-laminate structure was successfully snapped through from one stable shape to another. Efforts to model the shapes and behavior of the structure using the Rayleigh-Ritz technique showed promise: the shapes were closely predicted, but the snap-through voltage was under predicted. A possible significant reason for this under prediction may be nonlinearities, both electrical and mechanical, in the MFC actuator. Work is being done to try to quantify many of these nonlinearities.¹⁸

In the second part of the paper, the capability to model actuator-laminate structures was expanded to include finite-element modeling using ABAQUS. Two simple finite-element models were developed and compared with Rayleigh-Ritz solutions. Agreement between the finite-element and the Rayleigh-Ritz solutions was generally good: the predicted shapes were very similar, but the finite-element solutions were able to capture behavior not seen in the Rayleigh-Ritz solutions.

Future work will include finite-element modeling of more complicated actuator-laminate structures, and further experimental examination of this type of bi-stable structure.

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